

Two Fold eqn. of Two line not passing
through the origin

$$ax^2 + by^2 + 2hxy + 2fx + 2gy + c = 0$$

$$\Delta = \begin{vmatrix} a & h & f \\ h & b & g \\ f & g & c \end{vmatrix} = 0$$

equation of Bisectors for Two eqn not passing through (0,0)

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$

$$\tan \theta = \frac{2\sqrt{h^2 - ab}}{a + b}$$

EX:-

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2[ab+ac+bc]$$

$$y^2 - 6xy + 8x^2 - 2x - y - 6 = 0$$

prove that this represent Two Lines, Find them, Angle, Point of Intersection

$$\Delta = \begin{vmatrix} 8 & 3 & -1 \\ 3 & 1 & -\frac{1}{2} \\ -1 & -\frac{1}{2} & 6 \end{vmatrix} = 8(6 - \frac{1}{4}) - 3(18 - \frac{1}{2}) - 1(-\frac{3}{2} + 1) = 0 \quad \#$$

$$8x^2 - 6xy + y^2 = 0$$

$$(x-y)(4x-y) = 0$$

$$\frac{y}{x} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = 4x \quad \text{or} \quad y = 2x$$

$$y - 4x + \alpha = 0$$

$$y - 2x + \beta = 0$$

$$(y - 4x + \alpha)(y - 2x + \beta) = 0$$

$$\text{cfX: } [-4\beta - 2\alpha] = -2$$

$$\text{CoP y: } (\beta + \alpha) = -1$$

$$\text{Free Term: } \alpha\beta = 6$$

$$2\beta + 2\alpha = -2$$

$$-4\beta - 2\alpha = -2$$

$$-2\beta = -4$$

$$\beta = 2 \Rightarrow \alpha = 3$$

$$y - 4x + 3 = 0 \quad \text{or} \quad y - 2x + 2 = 0 \quad \#$$

$$\tan \theta = \frac{2\sqrt{9-8}}{1+8} = \frac{2}{9}$$

$$\theta = 12.5^\circ \quad \#$$

$$\frac{y - 4x + 3}{\sqrt{1+16}} = \frac{y - 2x + 2}{\sqrt{1+4}}$$

$$\frac{(y - 4x + 3)^2}{17} = \frac{(y - 2x + 2)^2}{5}$$

لي وجمع

Ex: Find Two P. & eqn. of Two lines that passing through $(2, 3)$
and parallel to the lines $2x^2 - 5xy + y^2 = 0$

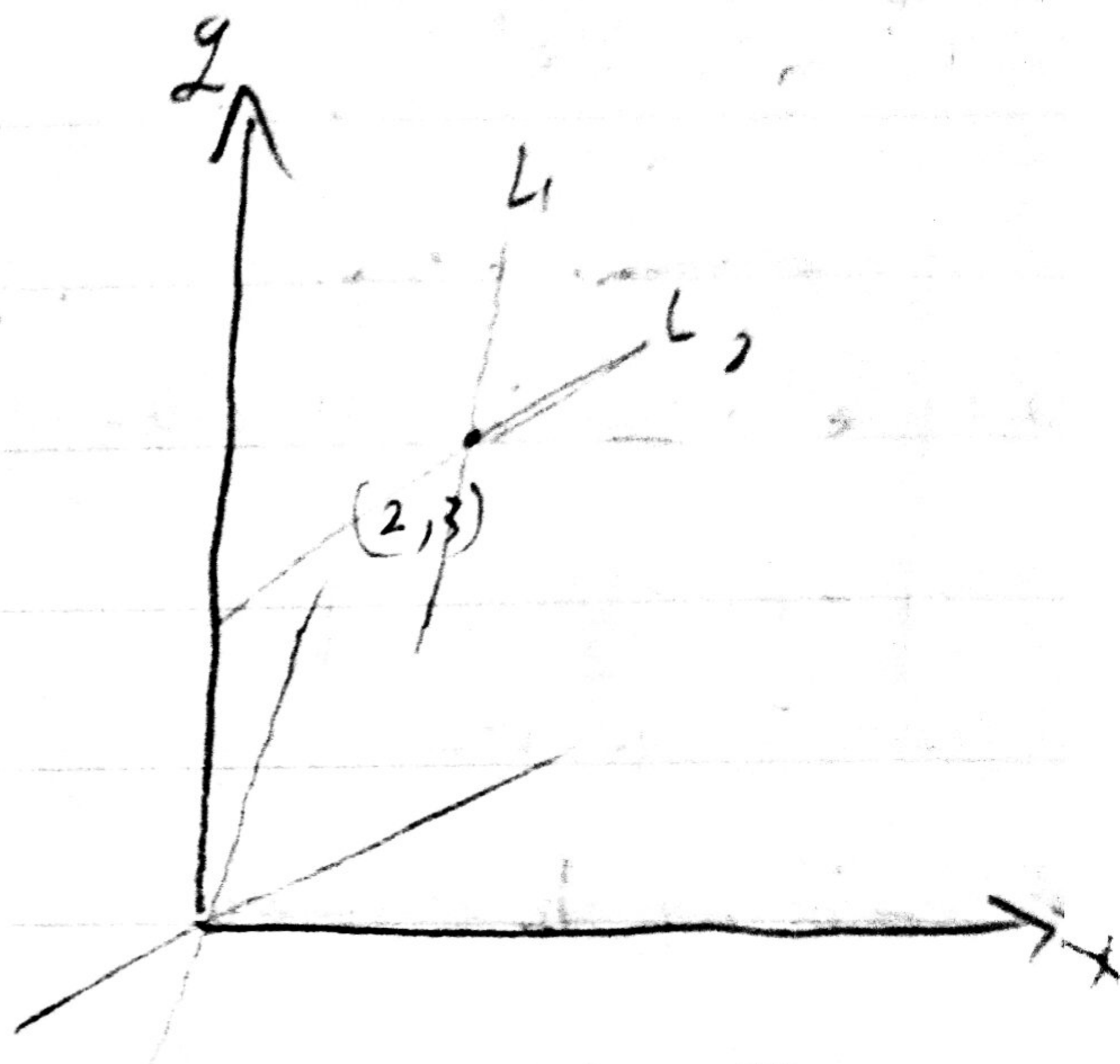
Then find the Bisectors

$$h = \frac{-5}{2}, a = 2, b = 1$$

$$\frac{y}{x} = \frac{-h \pm \sqrt{h^2 - ab}}{b}$$

$$\frac{y}{x} = 0.438$$

$$\frac{y}{x} = 4.563$$



$$L_1: y - 0.438x + \alpha = 0$$

$$L_2: y - 4.563x + \beta = 0$$

$$(2, 3)$$

$$3 - (2)(0.438) + \alpha = 0 \Rightarrow \alpha = -2.124$$

$$3 - (2)(4.563) + \beta = 0 \Rightarrow \beta = 6.126$$

CoF X [✓] CoF y [✓] Prefterm [✓]

$$2x^2 - 5xy + y^2 + 2Fx + 2gy + C = 0$$

$$\text{CoF X: } [-0.438\beta - 4.563\alpha]$$

$$\text{CoF y: } [\beta + \alpha]$$

$$\text{Prefterm: } \alpha/\beta$$

$$\tan \theta = \frac{2\sqrt{b^2 - ab}}{a+b} \Rightarrow \theta = 53.26^\circ$$

$$\frac{y - .438x - 2.124}{\sqrt{1^2 + (.438)^2}} = \frac{y - 4.563x + 6.25}{\sqrt{1^2 + (4.563)^2}}$$

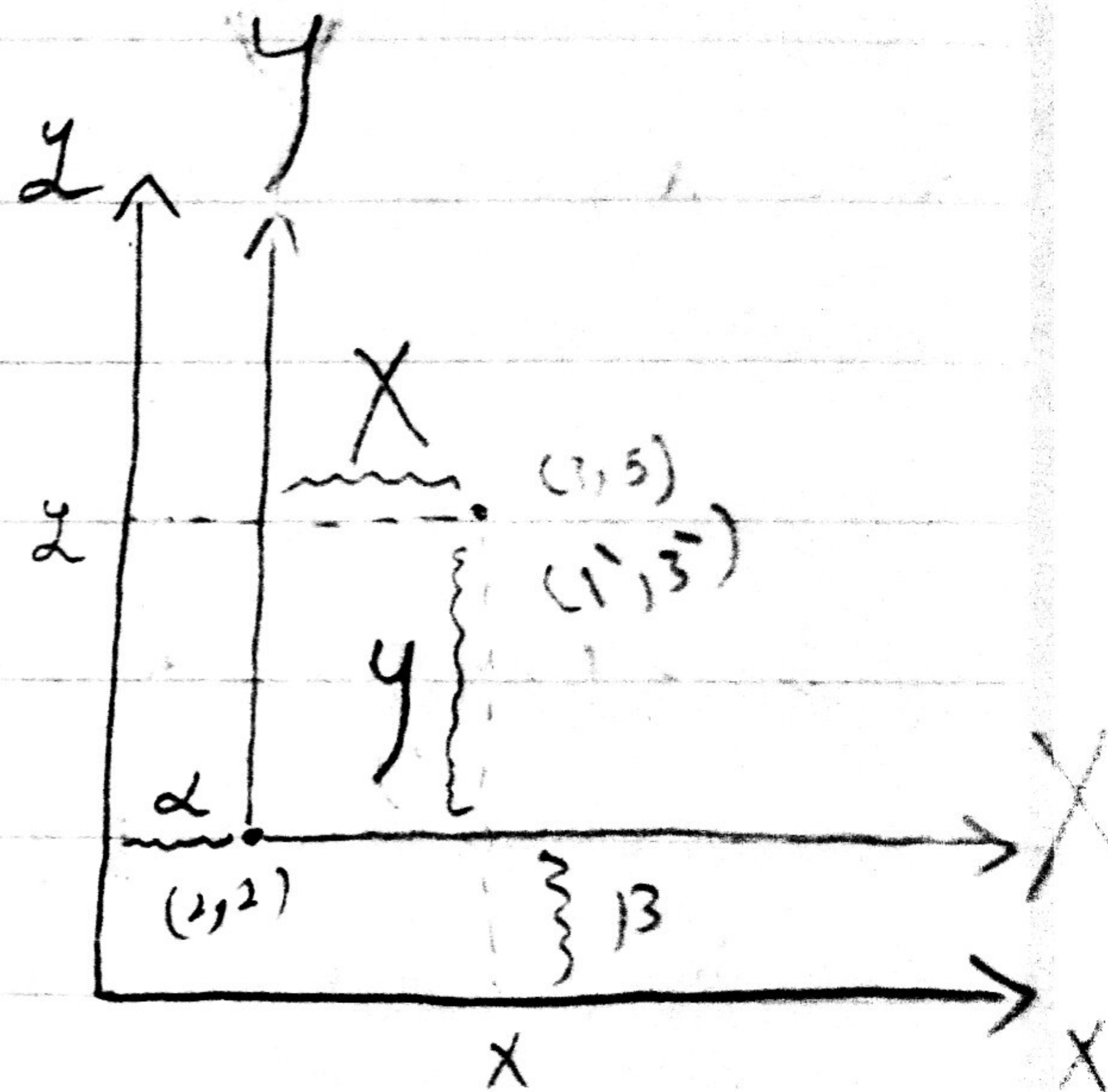
Translation of axes :-

$$x = X + \alpha$$

$$y = Y + \beta$$

$$(\alpha, 0) \Rightarrow x = X + \alpha, y = Y$$

$$(0, \beta) \Rightarrow x = X, y = Y + \beta$$



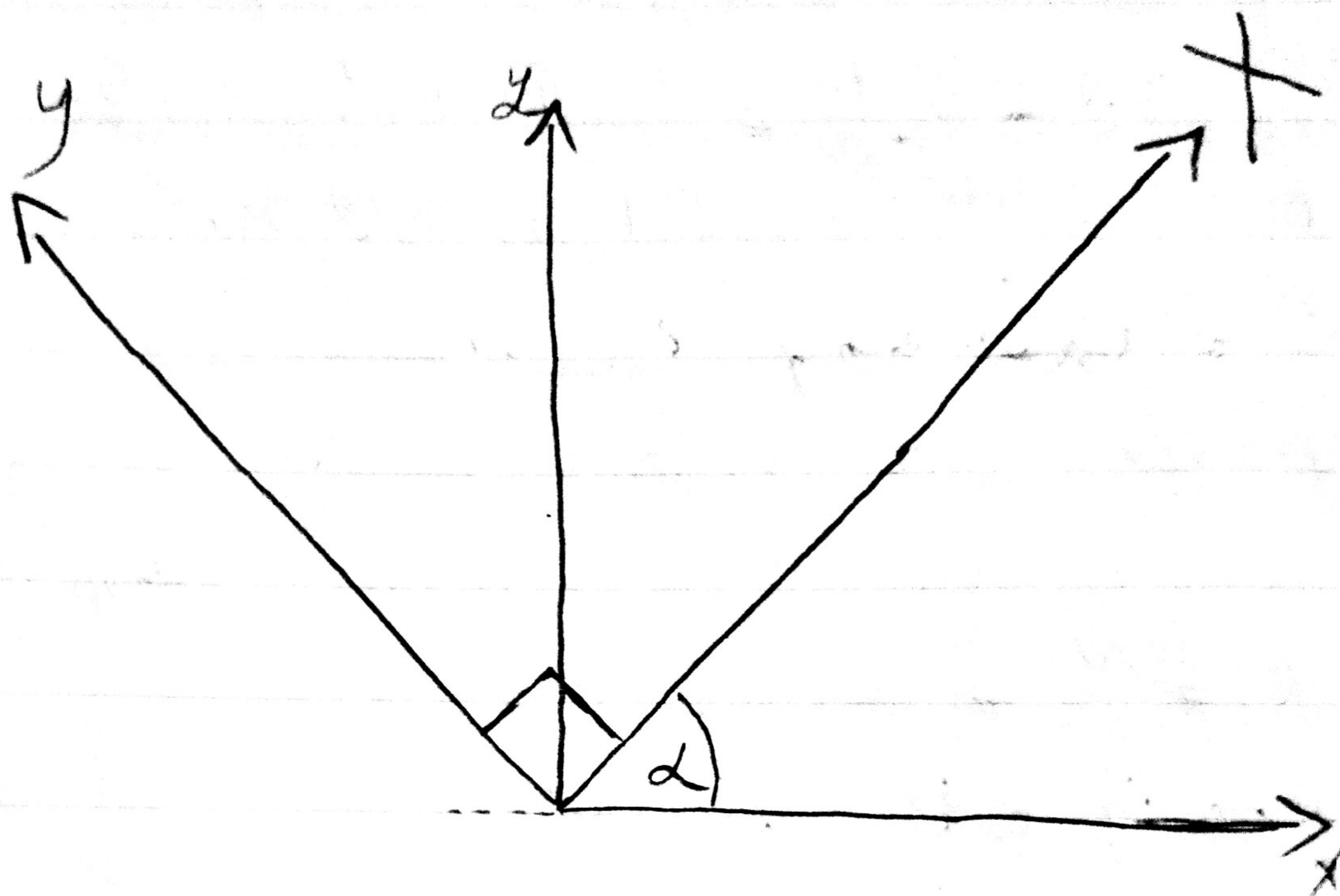
$$F \text{ is stationary} \rightarrow 1) \frac{dF}{dX} = 0, 2) \frac{dF}{dY} = 0$$

(1, 2) Solve $(\alpha, \beta) \rightarrow$ eliminate X, Y terms

Rotation of axes:-

X, y elimination

$$\tan \theta = \frac{2h}{a-b}$$



	x	y
x	$\cos \theta$	$\sin \theta$
y	$-\sin \theta$	$\cos \theta$

Ex: eliminate the first degree terms, then the term xy .

$$2x^2 + 5y^2 + 4xy - 4x - 22y + 7 = 0$$

$$\frac{dP}{dx} = 4x + 0 + 4y - 4 = 0$$

$$\frac{dP}{dy} = 10y + 4x - 22 = 0$$

Solve

$$(\alpha, \beta) = (-2, 3)$$

$$x = X - 2$$

$$y = Y + 3$$

$$2(X-2)^2 + 5(Y+3)^2 + 4(X-2)(Y+3) - 4(X-2) - 22(Y+3) + 7 = 0$$

$$2X^2 + 5Y^2 + 4XY + C = 0$$

$$C = f(\alpha, \beta) = f(-2, 3) = -22$$

$$2X^2 + 5Y^2 + 4XY - 22 = 0$$

$$\tan(2\theta) = \frac{4}{2-5} = \frac{-4}{3}$$

$$\theta = 26.56^\circ$$

$$X = X \cos \theta - Y \sin \theta$$

$$Y = X \sin \theta + Y \cos \theta$$

$$X = X \cos 26.56 - Y \sin 26.56$$

$$Y = X \sin 26.56 + Y \cos 26.56$$

$$2(X \cos \theta - Y \sin \theta) + 5(X \sin \theta + Y \cos \theta)$$

$$+ 4(X \cos \theta - Y \sin \theta)(X \sin \theta + Y \cos \theta)$$

$$- 22 = 0$$